Department of Mathematical Sciences
Sabbatical Presentation

"Nonlocal loss of first homotopy in polyhedral approximations of Peano continua"
Dr. Hanspeter Fischer

Thursday, April 17
1:00-1:50 p.m.
RB 450

Abstract:

The process of approximating topological spaces with finite polyhedra at progressively smaller scales is an important tool in many areas of mathematics, ranging from shape theory to persistent homology. One goal is to calculate algebraic invariants across a sequence of approximating polyhedra in order to draw conclusions about the underlying space. Common obstacles include local geometric features with global impact. Therefore, such methods work best for path-connected, locally path-connected, compact metric spaces, i.e., Peano continua.

If a Peano continuum X is semilocally simply connected, then it has a finite polyhedral approximation whose fundamental group is isomorphic to that of X. In general, this fails to be true. It is known that the fundamental group of a locally complicated Peano continuum may contain nontrivial elements that are persistently undetectable by polyhedral approximations, at all scales. In all known examples, this is due to one of the following local failures:

(1) Point-local failure: There is a point x in X and a nontrivial fundamental group element g such that for every neighborhood U of x, g can be represented by a finite product of path-conjugates of loops that lie in U.

(2) Path-local failure: There is an essential loop in X with inessential loops arbitrarily close to it.

However, we show that the failure is not inherently local. Specifically, we present an example of a 2-dimensional Peano continuum in Euclidean 3-space, whose fundamental group contains uncountably many elements that are undetectable by polyhedral approximations, while satisfying neither (1) nor (2). This is joint research with Jeremy Brazas (West Chester University of Pennsylvania).