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Design of Non-Uniform Linear Antenna Arrays Using Dolph-Chebyshev and Binomial Methods

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ABSTRACT
This paper explores the analytical methods of synthesizing linear antenna arrays. The synthesis employed is based on non-uniform methods. In particular, the Dolph-Chebyshev and binomial methods are used, so as to improve the directivity of the array and to reduce the level of the secondary lobes by adjusting the geometrical and electric parameters of the array. The radiation patterns, the directivity, and the array factors of the uniform and the non-uniform methods are presented. It is shown that the Chebyshev arrays have better directivity than binomial arrays for the same number of elements and separation distance, while binomial arrays have very low side lobes compared with Chebyshev and uniform excitation arrays. Finally, numerical results of both methods are analyzed and compared.
Keywords – array factor, directivity, radiation pattern, synthesis.

I. Introduction

Over the past few decades, since the concept of using antenna arrays instead of a single element has been developed, researchers have taken on the challenge of providing various array designs to tailor radiation characteristics according to system requirements [1]. Synthesizing an array depends on several factors, such as the requirements of the radiation pattern, the directivity pattern, etc. The radiation pattern depends on the number and the type of elements being used, and the physical and electrical structure of the array. Numerous variations of antenna structures, as well as the type of elements are available, but for simplicity only one kind of element is used in the whole array structure [2]. In other words, an antenna array is composed of an assembly of radiating elements in an electrical or geometrical configuration and, in most cases, the elements are identical (Fig. 1). The total field of the antenna array is found by vector addition of the fields radiated by each individual element. Five controls in an antenna array can be used to shape the pattern properly: the geometrical configuration of the overall array, the spacing between the elements, the excitation amplitude of the individual elements, the excitation phase of the individual elements, and the particular pattern of the individual elements [2-5].

Many communication applications require a highly directional antenna. Array antennas have higher gain and directivity than an individual radiating element. A linear array consists of elements placed in a straight line with a uniform spacing between the elements [6]. The goal of antenna array geometry synthesis is to determine the physical layout of the array which produces a radiation pattern that is closest to the desired pattern [5].

For the synthesis of the radiation pattern of antenna arrays, various analytical and numerical methods of optimization (End-Fire, Broadside, Hansen-Woodyard, binomial, Dolph-Chebyshev, Neural, Genetic, etc.…) were developed and applied [5-10]. Here, our focus is related to the various analytical methods. In particular, the non-uniform Dolph-Chebyshev and binomial methods will be applied to the synthesis of linear antenna arrays.

In this paper, we investigate the radiation pattern, the beam-width at half-power, the array factor and the directivity of the array. In addition, these parameters are comparatively investigated for uniform and non-uniform methods. Finally, we make a more detailed analysis of the influence of the number of elements and the spacing or distance between the elements on the main characteristics of the antennas. We also compare numerical results from all of the different analytical methods.

The paper is organized as follows: In Section II, we introduce the problem formulation. Section III describes the Dolph-Chebyshev method. Section IV describes the binomial method. Numerical results are presented and analyzed in Section V, and finally, section VI is devoted to the conclusion.

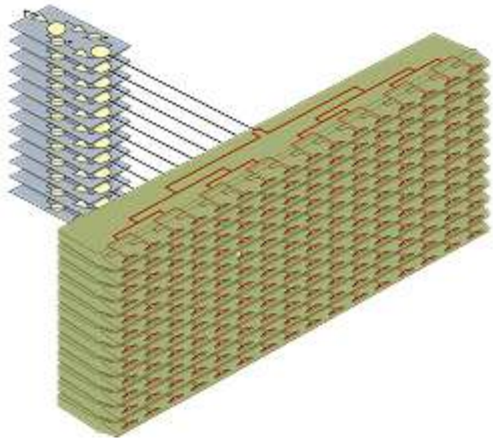

Figure 1. Sketch of a linear antenna array.

II. Problem formulation

Let us consider a linear array of N - elements aligned along the $z-$axis, at equal separation d from one another, as the diagram of Fig. 2 shows. The expression of the array factor for this case is given by the relation:

$$AF=\sum_{n=0}^{N-1} I_n Z^n, \quad (1)$$

where I_n is the amplitude of the current excitation of the n^{th} element, $Z=e^{j\psi}$, $\psi=kd\cos(\theta)+\beta$, θ is the angle between the radiation direction $\mathbf{r_n}$ and the axis of the array, $k=2\pi/\lambda$ is the wave number, λ is the wavelength, and β is the progressive phase difference between the elements.

For a symmetrical distribution of the current excitations, equation (1) becomes:

$$|AF|=2\sum_{n=1}^{N/2} I_n \cos\left(\left(n-\frac{1}{2}\right)\psi\right). \quad (2)$$

For an even number of elements, this array factor can be written as:

$$|AF|=I_0+2\sum_{n=1}^{(N-1)/2} I_n \cos(n\psi). \quad (3)$$

In equation (3), I_0 represents the current excitation of the central element of the array. If the spacing between elements is not identical, the array is known as a linear non-uniform array, and this condition forces a modification of the array factor in equations (2) and (3). However, the correspondence between the array factor of linear uniform array of N - radiating elements (equations (2) and (3)) and the non-uniform one is carried out either by using the binomial method or with Chebyshev polynomials of order $(N-1)$. By matching similar coefficients we obtain the excitation currents, I_n, required. In such cases, the array factors will be written in the following way:

$$AF=\sum_{n=1}^{M} I_n \cos[(2n-1)u], \quad (4)$$

for an even number of sources, and

$$AF=\sum_{n=1}^{M+1} I_n \cos[(2n-1)u], \quad (5)$$

for an odd number of sources, where $u=\frac{\pi d}{\lambda}\cos\theta$.

If we use a binomial expansion to determine these coefficients, the array is known as non-uniform binomial. On the other hand, if it utilizes Chebyshev polynomials, we refer to it as a non-uniform Dolph-Chebyshev array. For each array, it is necessary to determinate the array factor, the radiation pattern, the directivity ($D(\theta)=\frac{4\pi U(\theta)}{P_r}$, where P_r is the total radiated power, and $U(\theta)=\left(\frac{|AF(\theta)|}{max|AF(\theta)|}\right)^2$), and the beam-width at half-power.

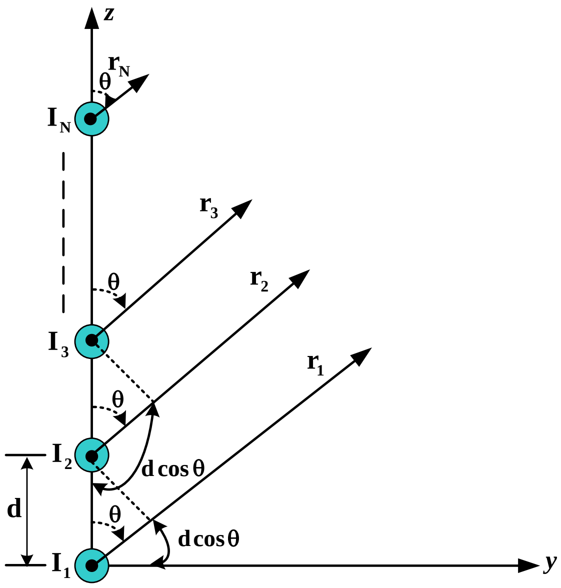

Figure 2. Geometry of the problem, showing a linear antenna array of N sources.

III. Binomial array

The general problem of the synthesis of arrays is similar to the problem of the synthesis of filters in the theory of linear circuits, with the angle θ replacing the frequency. It should be noticed that if the array does not have axial symmetry, we need two angles, θ and ϕ, to define each direction of radiation, and then the problem becomes more complicated. But we will confine ourselves here to arrays with axial symmetry.

In theory, we give ourselves a whole set of specifications relating to the behaviour of the array factor according to the angle (a gauge), and then the mathematical problem consists of synthesizing a function, $AF(\theta)$, obeying this gauge. The idea of the binomial array is to ask if we can eliminate the secondary radiation lobes by using a different distribution function for the amplitudes of the excitation currents of the antenna array. Indeed, it is known that in the case of an equidistant array, the appearance of secondary lobes is inevitable when the number N of elements increases. Moreover, the level of these lobes never goes down below the limit of - 13.3 dB.

On the basis of the idea of Serguei A. Schelkunoff, which rests on a simple mathematical relation such that with the complex auxiliary variable, $\omega = e^{jkd\cos(\theta)}$, the array factor in the linear equidistant case becomes $AF(\theta) = \sum_{n=0}^{N-1} I_n e^{jnkd\cos(\theta)}$, where I_n are the excitation currents. $AF(\theta)$ has an amplitude and a phase, and becomes a polynomial of the complex variable ω, where the complex coefficients are the excitations of the elements. Assuming that for an initial array with only two elements,

$$AF(\theta) = 1 + e^{j\psi} = 1 + \omega,$$

it is seen that the complex polynomial $1+\omega$ is cancelled only if $\omega = -1$. Further, it is known that if a function is not null for a certain value of the variable, the successive powers of this function are always non-zero.

The idea is then to try to synthesize a radiation pattern given by:

$$AF(\theta) = (1+\omega)^{N-1} = \sum_{n=0}^{N-1} \binom{N-1}{n} \omega^n , \qquad (6)$$

where $\omega = e^{j\psi} = e^{j(kd\cos(\theta)+\alpha)}$. The solution is thus to give to the array elements excitation amplitudes, I_n , corresponding to the coefficients of the binomial theorem. These are known since the only directions of null radiation are those of the initial array with two elements. In this way, we can control the existence of the secondary lobes.

IV. Dolph-Chebyshev array

Dolph indicated a method based on properties of the Chebyshev polynomials, which gives the possibility of obtaining the maximum gain for a fixed level of the secondary lobes imposed. This method uses the fact that the optimal distribution of the sources amplitudes is the one which gives, for the expression of the radiated field by an alignment of N sources, the Chebyshev polynomials of the degree $(N-1)$. These polynomials always present a significant maximum level which corresponds to the maximum of the main lobe of radiation. In addition, the polynomials present a succession of maxima and minima of equal amplitudes, which correspond here to the secondary lobes. Thus, we will present the synthesis of the desired radiation pattern by using Chebyshev polynomials of degree 20, $T_{19}(x_0)$, which will correspond, in the Dolph method, to the radiation pattern of an alignment of 20 sources [7].

With this method, all the secondary lobes of the pattern have the same level, which can present disadvantages if we wish that the antenna ensures a certain protection against jammers far away from the axis of the maximum radiation. On the other hand, we can show that an array built according to this method always presents the maximum gain compatible with the selected level of the secondary lobes.

In practice, the calculation of the distribution of amplitudes will be made as follows: we fix the relationship, R_0, between the amplitude of the maximum field of the main lobe and that of the secondary lobes. Then R_0 allows the definition of a parameter x_0, by the formula:

$$R_0 = T_{N-1}(x_0) = \cosh\left[(N-1)\cosh^{-1}(x_0)\right],$$

by taking into account that:

$$x_0 = \cosh\left[\frac{\cosh^{-1}(R_0)}{N-1}\right].$$

V. Results and discussion

It is known that the radiation pattern of an array is the product of the radiation pattern of an isolated element multiplied by the array factor. The array factor translates the effect of the relative position and the excitation of the elements. Clearly, the array factor is the radiation pattern which would be obtained if all the elements of the array were isotropic sources.

Figure 3 presents normalized array factors of the Broadside (array with transverse radiation), End-Fire (array with transverse radiation), and Hansen-Woodyard considered as uniform arrays, i.e., with all the elements having identical excitation amplitudes. The array is composed of 11 sources with spacing between elements of 0.25λ. In the case of the Broadside array, the normalized array factor is directed towards $\theta_0 = 90^\circ$. In other words, there is no main maximum in other directions. We can note that obtaining this orientation requires a choice of the excitation function and the appropriate number of elements, taking into account the spacing between the elements (which must be lower than λ to avoid unmatched lobes). Research has shown that for a separation between elements equal to $\lambda/2$, the Broadside radiation will see the appearance of the maximum lobes**.** The End-Fire array presents the maximum radiation directed along the axis $(\theta_0 = 180^\circ)$. If the separation between elements is equal to $\lambda/2$, the End-Fire radiation exists simultaneously in the two directions : $\theta_0 = 0^\circ$ and $\theta_0 = 180^\circ$. Concerning the curve of the array factor for Hansen-Woodyard, we see that the first desired maximum depends on the progressive phase β. If $\beta = \mp kd$, the first desired maximum tends either towards 0° ($(\beta = -kd)$, or towards 180° $(\beta = kd)$. Several researchers have also shown that the Hansen-Woodyard array is only an extension of the End-Fire array.

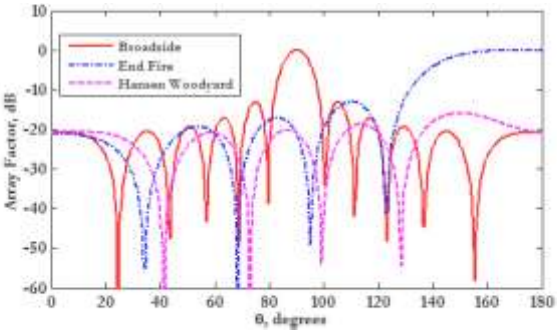

Figure 3. Normalized array factor of Broadside, End-Fire and Hansen-Woodyard arrays for $N = 11$ and $d = 0.25\lambda$. The curve for the array factor of Hansen-Woodyard shows that the first desired maximum depends on the progressive phase β. If $\beta = \mp kd$, the first desired maximum tends either towards 0° ($(\beta = -kd)$, or towards 180° $(\beta = kd)$.

Figure 4 presents the radiation patterns of Broadside, End-Fire and Hansen-Woodyard arrays for $N = 11$ and $d = 0.25\lambda$. As the results of simulations show, the Broadside array presents a narrow beam. As for the Hansen-Woodyard array, its conditions lead to a greater directivity than those given by End-Fire. However, we should specify that, these conditions do not necessarily bring back the desired maximum directivity. The maximum cannot even occur at $\theta_0 = 0^\circ$ or $\theta_0 = 180^\circ$, and research shows that the level of the secondary lobes cannot be greater than 13.46 dB. Clearly, the main maximum and the level of the secondary lobes depend on the number of elements of the array. Research also shows that for a rather large uniform array, the Hansen-Woodyard array can also improve directivity for spacing between the elements approximatively equal to $\lambda/4$.

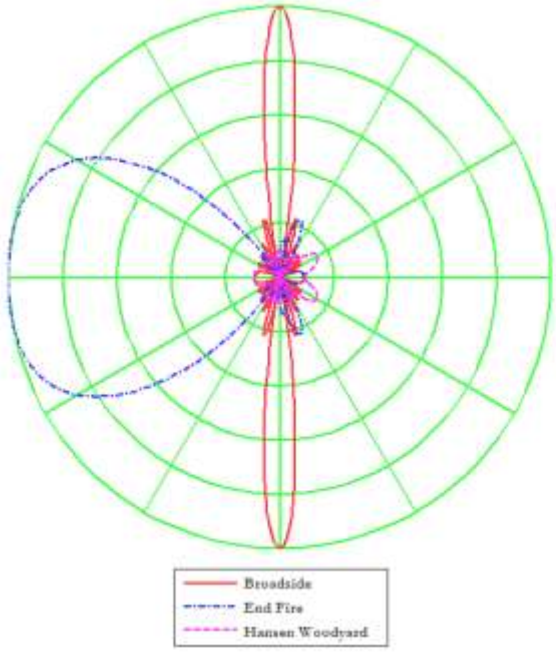

Figure 4. Radiation patterns of Broadside, End-Fire and Hansen-Woodyard arrays for $N = 11$ and $d = 0.25\lambda$. For a rather large uniform array, the Hansen-Woodyard array can also improve directivity for spacing between the elements approximatively equal to $\lambda/4$.

Tables 1 (a) and (b) show and analyze the radiation characteristics of the uniform arrays of Broadside, End-Fire and Hansen-Woodyard for linear antennas of 11 sources and various spacing between the sources. The tables show, (a) the level of the secondary lobes and the directivity, and (b) the beam-width at half power. We note that for the Broadside array, an increase in spacing between elements leads to degradation of the directivity and the level of the secondary lobes, while the beamwidth at half-power improves considerably. The best characteristics of radiation are obtained with $d = 0.25\lambda$. End-Fire and Hansen-Woodyard arrays present the best characteristics of radiation when $d = 0.25\lambda$ and $d = 0.55\lambda$, respectively. For the End-Fire and Hansen-Woodyard arrays, an increase in spacing between the elements also considerably improves the beam-width at half-power, but with certain variations when $d = 0.5\lambda$ for the End-Fire array and when $d = \lambda$ for the Hansen-Woodyard array. Thus, from the analysis of these uniform arrays, the data shows that the variation of spacing between elements makes it possible to improve at most two of the above-mentioned radiation-characteristics, but not more. In other words, it is difficult to optimize more than two characteristics at the same time. We also note that the improvement of one characteristic occurs in general at the detriment of another. Several researchers have shown that the directivity of a Hansen-Woodyard array is always approximatively 1.805 times (or 2.56 dB) larger than that of an ordinary End-Fire array [3], but our analysis, as shown in Table 1(a), indicates that this relationship does not uniformly apply.

Uniform linear arrays						
Spacing	Broadside (N=11)		End Fire (N=11)		Hansen Woodyard (N=11)	
d	D	SSLmax (dB)	D	SSLmax (dB)	D	SSLmax (dB)
0.25λ	5.648	-13.333	11	-5	19.594	5
0.35λ	7.804	-14.163	15.054	-4.916	25.077	5.097
0.45λ	9.941	-14.499	18.101	-4.332	21.053	27.834
0.5λ	11	-4.334	145.528	0	11	7.999
0.55λ	12.05	-4.006	8.327	10	11.157	-5.333
0.65λ	14.111	-3.996	9.606	30	12.933	7.999
0.75λ	16.076	-3.748	11	8.332	14.803	10.501
0.85λ	17.767	-3.663	12.372	8.123	16.616	11.178
0.95λ	17.591	6.667	13.510	8.000	17.534	11.987
λ	145.528	15.00	130.238	30	11	8.164

(a)

Uniform linear arrays			
Spacing	Broadside (N=11)	End Fire (N=11)	Hansen Woodyard (N=11)
d	HPBW (°)	HPBW (°)	HPBW (°)
0.25λ	18.605	66.067	36.803
0.35λ	13.260	55.610	31.066
0.45λ	10.304	48.935	27.379
0.5λ	9.272	0.108	25.935
0.55λ	8.427	44.201	12.542
0.65λ	7.129	40.620	8.077
0.75λ	6.177	37.788	6.422
0.85λ	5.450	35.477	5.492
0.95λ	4.876	33.543	4.876
λ	0.108	0.108	18.161

(b)

Table 1. Tables 1 (a) and (b) show and analyze the radiation characteristics of the uniform arrays of Broadside, End-Fire and Hansen-Woodyard for the linear antennas of $N = 11$ sources and various spacing between the sources. It is about (a) the level of the secondary lobes, of the directivity and (b) of the half-power of beam-width. It arises that the variation of spacing between elements makes it possible to improve one to two above mentioned characteristics of radiation but, not more. In other words, it is difficult at the same time to optimize more than two characteristics.

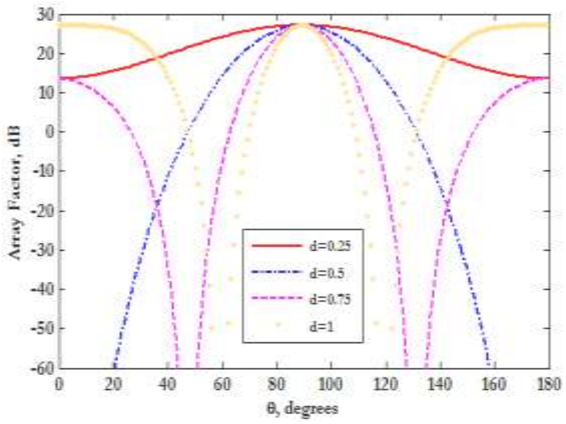

Figure 5. The normalized array factor of the binomial array of $N = 11$ sources, with spacing between elements varying from 0.25λ to λ. The binomial array with spacing between elements less than or equal to $\lambda/2$ does not have secondary lobes.

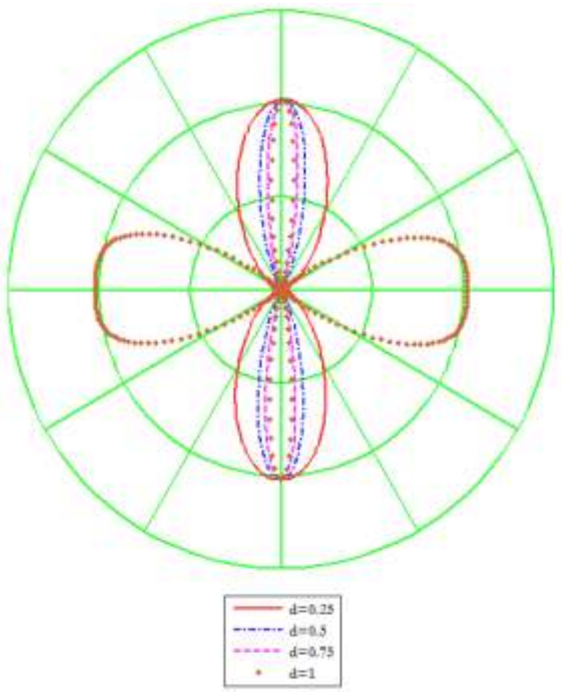

Figure 6. Radiation patterns of the binomial array of $N = 11$ sources, with spacing between elements varying from 0.25λ to λ. The binomial array usually has the smallest secondary lobes.

Figure 5 presents the normalized array factor of a binomial array of 11 sources, with spacing between elements varying from 0.25λ to λ. The analysis of the results shows that for a distance between elements less than or equal to a half-wavelength, the secondary lobes are nearly non-existent. In other words, a binomial array with spacing between elements less than or equal to $\lambda/2$ does not have secondary lobes. The secondary lobes appear when the spacing between elements is greater than 0.5λ. If the spacing reaches the value λ, the secondary lobes appear within 0° and 180°, in other words, as secondary maxima.

Figure 6 presents the radiation patterns of a binomial array of 11 sources with spacing between elements varying from 0.25λ to λ. The analysis of the curves shows that for a binomial array of antennas with spacing between elements less than a half-wavelength, the radiation pattern does not have secondary lobes. Thus, the array is less noisy (or at least its intrinsic temperature is lower) and we can detect weaker signals. For a spacing of greater than 0.75λ, we note the appearance of secondary lobes, as seen in Fig. 5. If we increase this spacing further to one wavelength, the radiation is not solely in Broadside but also in End-Fire. We can thus conclude that the optimal spacing for the binomial array is obtained when $0.1\lambda < d < 0.65\lambda$. Although the binomial distribution eliminates the zeros and small lobes (compare Fig. 6 with Fig. 4), the width of the beam emitted by the array decreases and, consequently the directivity improves. Let us notice that in the large-sized arrays, the amplitudes of the currents can vary considerably (according to the particular distribution pattern), which is likely to cause difficulty obtaining and preserving sufficiently stable power levels.

Tables 2 (a) and (b) present and analyze the radiation characteristics of the binomial and Dolph-Chebyshev arrays with odd and even numbers of elements, i.e., with 11 and 20 sources, and with spacing between sources varying from 0.25λ to λ. Table (a) shows that the best secondary lobe level for even and odd numbers of elements is obtained with $d = 0.75\lambda$. The directivity for either an even or odd number is also degraded with increased spacing between the elements. However, this trend reverses when $d = \lambda$. Lastly, the beam-width at half-power, as shown in Table (b), decreases with the growth of spacing between elements, but it dramatically increases at $d = \lambda$. The difference between the binomial array and the Dolph-Chebyshev is that Dolph-Chebyshev has a fixed secondary lobe level. In our case, its value is -20 dB.

spacing	Binomial N = 11		Binomial N = 20		Chebyshev N = 11		Chebyshev N = 20	
d	D	SSLmax (dB)	D	SSLmax (dB)	D	SSLmax (dB)	D	SSLmax (dB)
0.25λ	2.838	0	3.888	0	5.437	-20	10.011	-20
0.35λ	3.972	0	5.443	0	7.534	-20	13.735	-20
0.45λ	5.107	0	6.999	0	9.588	-20	17.308	-20
0.5λ	5.675	0	7.776	0	10.600	-20	19.041	-20
0.55λ	6.243	0	8.554	0	11.601	-20	20.740	-20
0.65λ	7.378	0	10.110	0	13.574	-20	24.042	-20
0.75λ	8.511	-36.636	11.665	-46.666	15.508	-20	27.228	-20
0.85λ	9.371	-13.333	13.183	-9.506	17.396	-20	30.307	-20
0.95λ	8.872	-8.1662	13.340	-11.838	17.022	-20	33.332	-20
λ	5.6755	-7328	7.777	4.664	10.600	-20	19.041	-20

(a)

Spacing	Binomial N = 11	Binomial N = 20	Chebyshev N = 11	Chebyshev N = 20
d	HPBW (°)	HPBW (°)	HPBW (°)	HPBW (°)
0.25λ	38.936	28.062	20.276	10.544
0.35λ	27.544	19.945	14.445	7.669
0.45λ	21.340	15.482	11.223	5.963
0.5λ	19.185	13.925	10.098	5.366
0.55λ	17.426	12.654	9.178	4.878
0.65λ	14.729	10.701	7.763	4.127
0.75λ	12.756	9.271	6.727	3.576
0.85λ	11.250	8.178	5.935	3.156
0.95λ	10.624	7.723	5.310	2.823
λ	47.109	40.102	34.121	24.841

(b)

Table 2. Tables 2 (a) and (b) present and analyze the radiation characteristics of the b3inomial and Dolph-Chebyshev arrays for 11 and 20 sources with spacing between sources varying from 0.25λ to λ. The difference between the binomial array and the Dolph-Chebyshev is that Dolph-Chebyshev has a fixed secondary lobe level.

The analysis of the two non-uniform arrays shows that for the same number of elements, the binomial array has the best results concerning the directivity and the level of the secondary lobes. The Dolph-Chebyshev array, on the other hand, has the best overall beam-width at half-power. As we mentioned above for the uniform arrays, it is difficult to concurrently optimize more than two characteristics of the radiation. Research has shown that the greatest disadvantage of a binomial array is the variation of the excitation amplitudes of the various elements of the array, especially when the number of elements is rather high.

The distribution technique which avoids the disadvantages discussed above is the Dolph-Chebyshev distribution. When we implement this distribution, it is necessary to specify the maximum level imposed on the secondary lobes if we want the minimum reduction of the width of the beam between first zeros, or inversely, it is necessary to specify the width of the beam between first zeros to reduce the secondary lobes to their minimum level.

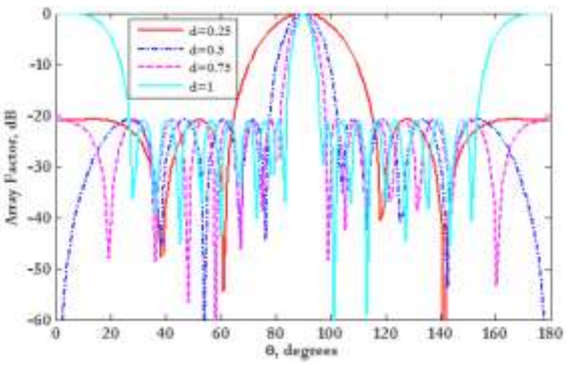

Figure 7. The normalized array factor of the Dolph-Chebyshev array for $N = 11$ with various spacings: $d = 0.25\lambda$, $d = 0.5\lambda$, $d = 0.75\lambda$ and $d = \lambda$. The desired secondary lobe level is fixed at -20 dB. We note that for a distance between the elements $d < \lambda$, we have only one maximum present at Broadside $(\theta = 90^\circ)$. As soon as this distance exceeds one wavelength $(d > \lambda)$, two other maxima also appear, at 0° and 180° (End-Fire radiation).

Figure 7 presents the normalized array factor of the Dolph-Chebyshev array for $N = 11$ with various spacings: $d = 0.25\lambda$, $d = 0.5\lambda$, $d = 0.75\lambda$ and $d = \lambda$. The desired level of secondary lobes is fixed at -20 dB. We note that for a distance between the elements $d < \lambda$, we have only one maximum present at Broadside $(\theta = 90^\circ)$. As soon as this distance exceeds one wavelength $(d > \lambda)$, two other maxima also appears, at 0° and 180° (End-Fire radiation). A progressive increase of this distance considerably decreases the width of the main lobe, thus improving the overall directionality of the array.

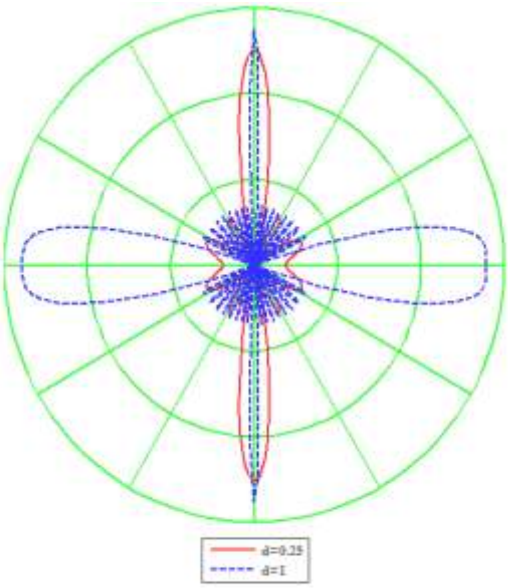

Figure 8. The radiation patterns of the Dolph-Chebyshev array for $N = 11$ with various spacings: $d = 0.25\lambda$ and $d = \lambda$.

Figure 8 presents the radiation patterns of the Dolph-Chebyshev array for $N = 11$ with two different spacings: $d = 0.25\lambda$ and $d = \lambda$. The directivity for Chebyshev is better than that of a binomial array for the same number of elements and separation distance. For a desired secondary lobe level, the beam-width at half-power and the directivity of the radiation patterns represented in Fig. 8 are given by using the following formula for approximating the widening of the array factor:

$$f = 1 + 0.636\left\{\frac{2}{R_0}\cosh\left[\sqrt{\left(\cosh^{-1} R_0\right)^2 - \pi^2}\right]\right\}^2, \qquad (7)$$

where R_0 is the ratio of the amplitude of the main lobe to the 1st secondary lobe, as defined earlier. The factor of beam-widening matches the diagram of figure 8 according to the level of the secondary lobes. The results clearly show by application of this formula that a larger value of R_0 leads to a considerable decreasing of the secondary lobe level. As in the case with the array factor, the desired radiation in Broadside, with a fixed level of secondary lobes equal to -20 dB for our case, is optimal only for one distance between elements $d < \lambda$. In contrast, however, two other secondary lobes will appear in the radiation pattern with the same amplitude as the main lobe in End-Fire.

VI. Conclusion

In this paper, we present the solution for the problem of synthesis of uniform and non-uniform linear antennas arrays. We determine the level of secondary lobes, the directivity, and the beam-width at half-power of each array. We can conclude by observing that:

- the directivity for Chebyshev is better than that of a binomial array for the same number of elements and separation distance,
- the optimal spacing for the Chebyshev array is $d = 0.95\lambda$, which gives the best radiation properties, while for the binomial array the optimal dimension is $d = 0.75\lambda$; the uniform array with spacing between the elements $d = 0.95\lambda$ has the best radiation properties,
- the binomial array has very low secondary lobes compared with Chebyshev and uniform excitation arrays because the coefficients of excitation of the binomial array are very large,
- the binomial array usually has the smallest secondary lobes, followed in order by the Dolph-Chebyshev array and the uniform arrays.
- for the three distributions, uniform, binomial, and Dolph-Chebyshev, the uniform distribution of amplitude achieves the smallest beam-width at half-power. Next in order comes the Dolph-Chebyshev array and the binomial array.

Finally, it is clear that for a better synthesis of antennas arrays, the designer must find a compromise between the level of the secondary lobes and the beam-width at half-power. The greatest challenge will consist in determining or choosing the values of the coefficients of excitation for a desired pattern of radiation.

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