Exam III review sheet for math 267

1. READ THE BOOK. Read every chapter of the covered material. You'll be amazed at how much you now know and this really will (trust me) solidify what you need to know. This seems like a lot of work, but it should move quickly as the beginning material will be second nature to you by now. But you still need to re-read it since now you can move past mere proficiency and actually understand why you did all those calculations before.

2. Re-do all the homework and quiz problems (don't just look over your old solutions).

3. Do the following make sense? If so, what do they represent?

(a) $\int_0^x \int_1^2 \int_0^4 1\, dx\, dy\, dz$

(b) $\int_0^4 \int_1^2 \int_0^x 1\, dz\, dy\, dx$

(c) $\int_0^4 \int_0^x \int_1^2 1\, dy\, dz\, dx$

(d) $\int_0^4 \int_0^z \int_1^2 1\, dy\, dx\, dz$

(e) $\int_1^2 \int_0^x \int_0^4 1\, dx\, dz\, dy$

4. How do we transform **equations** in cylindrical coordinates to spherical coordinates? cylindrical coordinates to rectangular (Cartesian) coordinates? spherical coordinates to rectangular (Cartesian) coordinates?

5. How do we transform **integrals** from rectangular (Cartesian) coordinates to spherical coordinates? to cylindrical coordinates? to arbitrary coordinates?

6. What is the Jacobian? What do we use it for?

7. **Using spherical coordinates** find the volume of the region above the graph of $z = \sqrt{x^2 + y^2}$ (cone) and below the graph of $z = \sqrt{32 - x^2 - y^2}$ (hemisphere).

8. Consider the area A of the region of the ellipse $\frac{x^2}{9} + y^2 = 1$ that lies in the first quadrant. The line $y = mx$ divides this region into two areas A_1 and A_2.

(a) Make a change of coordinates that will change the ellipse into a circle.

(b) Graph the regions B_1 and B_2 in the new coordinate system which correspond to A_1 and A_2 in the old coordinate system. Write down the equation of the boundary between B_1 and B_2.

(c) Represent the areas A_1 and A_2 as integrals over B_1 and B_2 in the new coordinate system.

(d) Find the value of m that makes the areas A_1 and A_2 equal. Hint: Evaluate the integrals by considering the relationship between the integrals and the areas over which you are integrating.

9. Let $D = \{(x, y, z) | 1 \le z \le 2, 1 \le x^2 + y^2 \le 2\}$. Convert the integral $\iiint_D z\sqrt{x^2 + y^2}\, dV$ into an iterated integral using a suitable choice of coordinates and evaluate this integral.

10. Consider the solid E which is bounded by the sphere $x^2 + y^2 + z^2 = 9$ and the cone $z = \sqrt{x^2 + y^2}$. Let $f(x, y, z) = x + y + z$ and write, **but do not evaluate** $\iiint_V f(x, y, z)\, dV$ as an iterated triple integral in:
    (a) Rectangular coordinates (b) Spherical coordinates (c) cylindrical coordinates

11. Consider the region R bounded by the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$. Evaluate $\displaystyle\iint_R x^2 + \frac{9y^2}{4}\, dA$. (Hint: make a change of coordinates.)

12. Evaluate $\displaystyle\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{1}^{16-x^2-y^2} \frac{1}{z^2}\, dz\, dy\, dx.$

13. How can you tell just by looking at a plot of the vector field $(P, Q, 0)$ whether or not the curl is the zero vector? How can you tell just by looking at a vector field $(P, Q, 0)$ whether or not the divergence is zero?

14. What is a potential function? How do we use it? How do we find one? Can we always find one?

15. What is independence of path? Is the line integral of a gradient field always path independent? Are the line integrals of all vector fields always path independent?

16. What is independence of parametrization? Is the line integral of a gradient field always independent of parametrization? Are the line integrals of all vector fields always independent of parametrization?

17. When is a line integral positive? Negative? Zero? How do we actually evaluate them?

18. Carefully state Green's Theorem. When should we use Green's Theorem?

19. Evaluate $\int_C y^3\, dx - x^3\, dy$ where C is the unit circle traversed CCW.

20. Evaluate $\int_C (y^3 + \frac{\sqrt{x+95}}{e^x \ln(x+123)})\, dx - x^3\, dy$ where C is the unit circle traversed CCW.

21. Graph the vector field **F**=x **i**+ y **j**. Does this vector field have a potential function? If so, find it.

22. Graph the vector field **F**=-y **i**+ x **j**. Does this vector field have a potential function? If so, find it.

23. Graph the vector field **F**=-y **i**+ x **j**. Let C be the positively oriented curve that traverses the boundary of the triangle formed by the points (0,0), (1,0) and (0,3). Explain why $\int_C \mathbf{F} \cdot d\mathbf{X} = \int_{C'} \mathbf{F} \cdot d\mathbf{X}$ where C' is the line segment running from (1,0) to (0,3). Is this line integral positive, negative or zero?

24. Say that **F** is a vector field. Do the following make sense? If so, are they vectors or scalars?
    (a) div(curl **F**) (b) curl(div **F**) (c) ∇(curl **F**)
    (d) $\nabla\cdot$ (curl **F**) (e) $\nabla\times$ (curl **F**) (f) ∇ (div **F**)