Math 267

Review Sheet II

• READ THE BOOK. Re-read every chapter of the covered material.

• Re-do all the homework problems (don't just look over your old solutions).

• Learn all the necessary formulas, but be sure to be able to get by as well as you can without them.

• AFTER doing the above then work on this Exam II Review sheet problems.

1. What is the gradient of a function? What are its properties? What do we do with it? What are the relevant formulas?

2. How do we find critical points and classify them?

3. Find the equation of the tangent plane to the surface
$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$$
at the point (1,2,3) in three different ways described below.

   a) Solve for z to get a function $z = f(x, y)$ and then proceed by using the equation
$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

   b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ **implicitly** and then use the equation
$$z - z_0 = \left.\frac{\partial z}{\partial x}\right|_{(x_0,y_0)}(x - x_0) + \left.\frac{\partial z}{\partial y}\right|_{(x_0,y_0)}(y - y_0).$$

   c) Consider the surface as the level surface of $g(x, y, z) = x^2 + \frac{y^2}{4} + \frac{z^2}{9}$ and use ∇g to find a normal vector to the surface.

4. Use linear approximation to estimate $e^{2.1} + \sin(0.2)$.

5. Use differentials to estimate $e^{2.1} + \sin(0.2)$.

6. Use Lagrange Multipliers to find the max/min of $f(x, y) = 4x + 6y$ for $x^2 + y^2 = 13$.

7. How can you use the tree method to find the chain rule for a given situation?

8. Let $f(x, y, z)$ and $g(x, y)$ be functions and let $\vec{a}, \vec{b}$ and $\vec{c}$ be 3-d vectors and $\vec{u}$ is a 3-d unit vector. Are the following scalars (numbers), vectors, or do they just not make sense?

(a) $\nabla f(x, y, z) \times \vec{a}$

(b) $\nabla f(x, y, z) \cdot \vec{a}$

(c) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

(d) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(e) $D_{\vec{u}} f(x, y, z) \times \nabla f(x, y, z)$

(f) $\vec{u} \times \nabla f(x, y, z)$

(g) $|\vec{a} \times \vec{b}| \cdot \nabla f(x, y, z)$

(h) $|\vec{a} \times \vec{b}| \times \nabla f(x, y, z)$

(i) $D_{\vec{u}} f(x, y, z) \cdot \vec{u}$

(j) $\nabla f \cdot \nabla g$

(k) $|\nabla f(x, y, z)| + D_{\vec{u}} f(x, y, z)$

9. Let $F(x, y, z) = x^2 + \frac{y^2}{4} + \frac{z^2}{9}$. I'm standing at the point (1,2,9) and shoot a bullet at a target which sits at the point (5,4,13). If I shoot in the direction of ∇F do I hit the target?

10. How do we use Riemann sums to approximate a double integral over a rectangle R?

11. Write out the chain rule for each independent variable given $v = f(p, q, r)$ $\quad p = p(x, y, z)$
$q = q(x, y, z)$ $\quad r = r(x, y, z)$.

12. Use the chain rule to find $\partial w/\partial t$ and $\partial w/\partial s$ at $s = 1, t = 0$ where
$w = x^2y^2 + yz^2, \quad x = st, \quad y = s\cos t, \quad z = s\sin t.$

13. Find and classify the critical points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

14. Estimate $\iint_R \sin y \, e^x \, dA$ by calculating a Riemann sum over four rectangles where R is the rectangle $0 \le x \le 2, 0 \le y \le 4$.

15. Evaluate $\iint_R \sin y \, e^x \, dA$ for R above.

16. How do we maximize and/or minimize a function with a constraint?

17. What is the method of Lagrange Multipliers? What does it mean when the constraint curve is bounded? What does it mean when the constraint curve is unbounded? What does the Lagrange Multiplier equation mean geometrically?

18. Sketch the region of integration for $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{x+3} x^3\, dy\, dx$.

19. **Using polar coordinates** find the volume of the region between the graphs of $z = \sqrt{x^2+y^2}$ (cone) and $z = \sqrt{32 - x^2 - y^2}$ (hemisphere).

20. Evaluate $\int_0^1 \int_y^1 \frac{1}{1+x^2}\, dx\, dy$.

21. A pizza centered at the origin of the xy-plane has coordinates of its perimeter defined by $x^2 + y^2 = 64$, and the thickness of the pizza is given by $T(x, y) = \frac{\sqrt{x^2+y^2}+12}{20}$. Find the volume of the pizza.

22. Find the center of mass of the triangular region with vertices (0,0), (1,1), and (4,0) with density function $\rho(x) = x$.

23. The contour diagram of a function $f(x, y)$ appears as Diagram A at the end of this sheet. In part (a) - (h) determine whether the object being considered is positive negative or zero at **each** of the points P and Q.

(a) $\nabla f \cdot \frac{\vec{i}+\vec{j}}{\sqrt{2}}$

(b) $\frac{\partial f}{\partial x}\frac{1}{\sqrt{2}} - \frac{\partial f}{\partial y}\frac{1}{\sqrt{2}}$

(c) $\nabla f \cdot \vec{i}$

(d) $\frac{\partial f}{\partial x}$

(e) $D_{\vec{u}} f$ where $\vec{u} = \frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$

(f) $\frac{\partial f}{\partial y}$

(g) $\nabla f \cdot \vec{j}$

(h) $D_{\vec{u}} f$ where $\vec{u} = \frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$

(i) Which of the items in (a)-(h) are equal? Why?