Math 267

Review Sheet 1

1. Given the points $A(5,5,1)$, $B(3,3,2)$ and $C(1,4,4)$, determine whether triangle ABC is isosceles, right, both, or neither.

2. Find the equation of the sphere with center $C(2,0,-5)$ and radius $\frac{1}{3}$.

3. Find the center and radius of the sphere

   $x^2+y^2+z^2+x+2y-6z-2=0$.

4. Describe the following regions in R^3:

   a. $y=8$

   b. $z>0$

   c. $x^2+y^2+z^2 \geq 25$

   d. $xz=0$

   e. $x^2+y^2 \leq 8$

5. Given $\mathbf{a} =< 1,5,7 >$, $\mathbf{b} =< 2,0,5 >$, find $\mathbf{a}-\mathbf{b}$, $\mathbf{a}+\mathbf{b}$, and $2\mathbf{a}-\frac{1}{2}\mathbf{b}$ and illustrate each one graphically.

6. Find a unit vector that has the same direction as $\mathbf{a} =< -3,8,5 >$

7. Find the angle between the vectors $\mathbf{a} = 3\mathbf{i}+2\mathbf{j}$ and $\mathbf{b} = 5\mathbf{j}+\mathbf{k}$.

8. Find the angle α in $[0,\pi]$ that the vector

   $\mathbf{a} =< 3,5,-4 >$ makes with the positive x axis.

9. Find the value(s) of x so that the vectors

   $< x,x,-1 >$ and $< 1,x,6 >$ are orthogonal.

10. Compute the the cross product of the vectors

    $\mathbf{a} =< -8,-2,-4 >$ and $\mathbf{b} =< 2,2,1 >$.

11. Find two unit vectors orthogonal to both

    $< 1,1,0 >$ and $< 1,-1,1 >$.

12. Given the points $P(1,-1,2)$, $Q(2,0,-1)$ and $R(0,2,1)$, find a vector perpendicular to the plane containing these points and find the area of the triangle determined by these points.

13. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} =< 2,3,-2 >$,

    $\mathbf{b} =< 1,-1,0 >$ and $\mathbf{c} =< 2,0,3 >$.

14. Find parametric and symmetric equations for the line passing through the points $(1,2,4)$ and $(2,-1,5)$.

15. Find parametric equations for the line passing through $(5,1,0)$ that is perpendicular to the plane $2x-y+z=1$. At what point does this line intersect the xz plane?

16. Determine whether the given lines are parallel, intersecting, or skew.

    L1: $x=1+t$, $y=2-t$, $z=3t$

    L2: $x=2-s$, $y=1+2s$, $z=4+s$.

17. Find the equation of the plane passing through $(1,-1,3)$ and parallel to $3x+y+z=7$.

18. Find the equation of the plane passing through $(1,1,-1)$, $(2,0,2)$ and $(0,-2,1)$.

19. Find the equation of the plane passing through $(2,4,5)$ and perpendicular to the line $x=5+t$, $y=1+3t$, $z=4t$.

20. Find the equation of the plane passing through $(-1,0,1)$ that contains the line $x=5t$, $y=1+t$, $z=-t$.

21. Find an equation of the line of intersection of the planes $x-2y+4z=2$ and $x+y-2z=5$.

22. Find the angle between the planes $x+y=1$ and $2x+y-2z=2$.

23. Where does the line $x=1+t$, $y=2t$ and $z=3t$ intersect the plane $x+y+z=1$?

24. Find the unit tangent vector for the equation $\mathbf{r}(t) =< e^t, e^{2t}, te^{3t} >$ at the point $(1,1,0)$.

25. Find parametric equations for the tangent line to the curve $< 1+2t, 1+t-t^2, 1-t+t^2-t^3 >$ where $t=0$.

26. Find $\mathbf{r}(t)$ given that

    $\mathbf{r}'(t) =< \sin t, -\cos t, 2t >$ and

    $\mathbf{r}(0) =< 1,1,2 >$.

27. The curves $\mathbf{r_1}(t) =< t, t^2, t^3 >$ and

    $\mathbf{r_2}(t) =< \sin t, \sin 2t, t >$ intersect at the origin. Find their angle of intersection.

28. How would you distinguish the graphs of the following?

a. $6x + 4y + 2z = 7$
b. $3x^2 + y^2 + z^2 = 1$
c. $z = x^2 + 4y^2$
d. $z^2 = 4y^2 - 9x^2$

29. Give the equation of a quadric surface whose traces perpendicular to the x-axis are circles and whose traces perpendicular to the z-axis are parabolas.

30. Find the arclength of the curve given by $\vec{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle$ where $0 \leq t \leq 2$.

31. Sketch levels curves and the graph of the function.

a. $f(x, y) = 1 - x^2 - y^2$
b. $g(x, y) = \sqrt{x^2 + y^2 - 1}$
c. $h(x, y) = e^{-(x^2+y^2)}$

32. Find all first and second partial derivatives.

a. $f(x, y) = x^2y^3 - 2x^4 + y^2$
b. $g(x, y) = x^3 \ln(x - y)$
c. $h(x, y, z) = xe^y \cos z$
d. $k(x, y) = e^{xy}$