Homework 7 solutions

15.1 #1 (a) The subrectangles are shown in the figure.

The surface is the graph of $f(x, y) = xy$ and $\triangle A = 4$

so we estimate

$$V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f(x_i, y_j) \triangle A$$

$$= f(2,2)\triangle A + f(2,4)\triangle A + f(4,2)\triangle A + f(4,4)\triangle A + f(6,2)\triangle A + f(6,4)\triangle A$$

$$= 4(4) + 8(4) + 8(4) + 16(4) + 12(4) + 24(4) = 288$$

(b)

$$V \approx \sum_{i=1}^{3} \sum_{j=1}^{2} f(\bar{x}_i, \bar{y}_j) \triangle A$$

$$= f(1,1)\triangle A + f(1,3)\triangle A + f(3,1)\triangle A + f(3,3)\triangle A + f(5,1)\triangle A + f(5,3)\triangle A$$

$$= 1(4) + 3(4) + 3(4) + 9(4) + 5(4) + 15(4) = 144$$

15.1 #9 (a) With $m = n = 2$, we have $\triangle A = 4$. Using the contour map to estimate the value of f at the center of each subrectangle, we have

$$\int\int_R f(x, y)\, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f(\bar{x}_i, \bar{y}_j) \triangle A$$

$$= \triangle A[f(1,1) + f(1,3) + f(3,1) + f(3,3)] \approx 4(27 + 4 + 14 + 17) = 248$$

(b)

$$f_{ave} = \frac{1}{A(R)} \int\int_R f(x, y)\, dA \approx \frac{1}{16}(248) = 15.5$$

15.2 #7

$$\int_0^2 \int_0^1 (2x+y)^8 \, dx \, dy = \int_0^2 [\frac{1}{2}\frac{(2x+y)^9}{9}]|_{x=0}^{x=1} \, dy$$

$$= \frac{1}{18}\int_0^2 [(2+y)^9 - (0+y)^9] \, dy = \frac{1}{18}[\frac{(2+y)^{10}}{10} - \frac{y^{10}}{10}]|_0^2$$

$$= \frac{1}{180}[(4^{10}-2^{10}) - (2^{10}-0^{10})] = \frac{1,046,528}{180} = \frac{261,632}{45}$$

15.3 #18

$$\int \int_D 2xy \, dA = \int_0^1 \int_{2x}^{3-x} 2xy \, dy \, dx$$

$$= \int_0^1 [xy^2]_{y=2x}^{y=3-x} \, dx = \int_0^1 x[(3-x)^2 - (2x)^2]dx$$

$$= \int_0^1 (-3x^3 - 6x^2 + 9x) \, dx = [-\frac{3}{4}x^4 - 2x^3 + \frac{9}{2}x^2]_0^1$$

$$= -\frac{3}{4} - 2 + \frac{9}{2} = \frac{7}{4}$$

15.3 #47

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1} \, dy \, dx = \int_0^2 \int_0^{y^2} \frac{1}{y^3+1} \, dx \, dy$$

$$= \int_0^2 \frac{1}{y^3+1}[x]_{x=0}^{x=y^2} \, dy = \int_0^2 \frac{y^2}{y^3+1} \, dy$$

$$= \frac{1}{3}\ln|y^3+1||_0^2 = \frac{1}{3}(\ln 9 - \ln 1) = \frac{1}{3}\ln 9$$