Homework 3 solutions

13.1 #19-24 19-VI, 20-II, 21-IV, 22-I, 23-V, 24-III

13.2 # 25 The vector equation for the curve is $\vec{r}(t) = \langle e^{-t}\cos t, e^{-t}\sin t, e^{-t}\rangle$ So $\vec{r'}(t) = \langle e^{-t}(-\sin t) + (-e^{-t})\cos t, e^{-t}\cos t + e^{-t}(-\sin t), (-e^{-t})\rangle = \langle -e^{-t}(\sin t + \cos t), e^{-t}(\cos t - \sin t), -e^{-t}\rangle$. The point $(1, 0, 1)$ corresponds to $t = 0$, so the tangent vector there is $\vec{r'}(0) = \langle -e^0(\sin 0 + \cos 0), e^0(\cos 0 - \sin 0), -e^0\rangle = \langle -1, 1, -1\rangle$. Thus the tangent line is parallel to the vector $\langle -1, 1, -1\rangle$ and so has vector form $\vec{s}(t) = \langle 1, 0, 1\rangle + t\langle -1, 1, -1\rangle = \langle 1-t, t, 1-t\rangle$. Thus the parametric equations are

$$x = 1 - t \qquad y = t \qquad z = 1 - t.$$

13.2 # 31 The angle between the curves $\vec{r_1}(t) = \langle t, t^2, t^3\rangle$ and $\vec{r_2}(t) = \langle \sin t, \sin 2t, t\rangle$ will be the angle between the corresponding tangent vectors at the point of intersection, which is when $t = 0$. Since $\vec{r_1'}(t) = \langle 1, 2t, 3t^2\rangle$ and $\vec{r_2'}(t) = \langle \cos t, 2\cos 2t, 1\rangle$ we can then use the dot product to find the angle θ,

$$\vec{r_1'}(0) \cdot \vec{r_2'}(0) = |\vec{r_1'}(0)||\vec{r_2'}(0)|\cos\theta$$

$$\langle 1, 0, 0\rangle \cdot \langle 1, 2, 1\rangle = (1)(\sqrt{6})\cos\theta$$

and so $\theta = \cos^{-1}(1/\sqrt{6})$.

13.3 # 3

$$L = \int_0^1 |\vec{r'}(t)|\,dt = \int_0^1 \sqrt{2 + e^{2t} + e^{-2t}}\,dt$$

$$= \int_0^1 \sqrt{(e^t + e^{-t})^2}\,dt = \int_0^1 e^t + e^{-t}\,dt = e^t - e^{-t}|_0^1 = e - e^{-1}.$$

13.3 # 14

$$\vec{r}(t) = e^{2t}\cos 2t\vec{i} + 2\vec{j} + e^{2t}sin2t\vec{k}$$

such that

$$\vec{r'}(t) = 2e^{2t}(cos2t - sin2t)\vec{i} + 2e^{2t}(cos2t + sin2t)\vec{k},$$

$$\frac{ds}{dt} = |\vec{r'}(t)| = 2e^{2t}\sqrt{(cos2t - sin2t)^2 + (cos2t + sin2t)^2}$$
$$= 2e^{2t}\sqrt{2cos^2 2t + 2sin^2 2t} = 2\sqrt{2}e^{2t}$$
$$s = s(t) = \int_0^t |\vec{r'}(u)|\, du = \int_0^t 2\sqrt{2}e^{2u}\, du = \sqrt{2}e^{2u}|_0^t = \sqrt{2}(e^{2t} - 1)$$

so

$$\frac{s}{\sqrt{2}} + 1 = e^{2t}$$
$$t = \frac{1}{2}\ln(\frac{s}{\sqrt{2}} + 1).$$

Substituting, we have

$$\vec{r}(t(s)) = e^{2(\frac{1}{2}\ln(\frac{s}{\sqrt{2}}+1))}cos2(\frac{1}{2}\ln(\frac{s}{\sqrt{2}} + 1))\vec{i} + 2\vec{j} + e^{2(\frac{1}{2}\ln(\frac{s}{\sqrt{2}}+1))}sin2(\frac{1}{2}\ln(\frac{s}{\sqrt{2}} + 1))\vec{k}$$
$$= (\frac{s}{\sqrt{2}} + 1)cos(\ln(\frac{s}{\sqrt{2}} + 1))\vec{i} + 2\vec{j} + (\frac{s}{\sqrt{2}} + 1)sin(\ln(\frac{s}{\sqrt{2}} + 1))\vec{k}.$$