Homework 2 solutions

§12.4 #15 Find $|\vec{u} \times \vec{v}|$ and determine whether is directed into or out of the page.

To determine the magnitude of $\vec{u} \times \vec{v}$, we first must determine the angle between the vectors. By moving the base of the vectors to the same point, it is clear that $\theta = 30^o$ since a straight line is 180^o. So,

$$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \sin(\theta) = 6 \cdot 8 \sin(30^o) = 24.$$

By the right hand rule, $\vec{u} \times \vec{v}$ is directed into the page.

§12.4 #30 (a) Find a vector orthogonal to the plane through the points $P(2, 1, 5)$, $Q(-1, 3, 4)$, and $R(3, 0, 6)$.

If a vector is orthogonal to the plane through the given points, then it must be orthogonal to both the vectors $\vec{PQ} = \langle -3, 2, -1 \rangle$ and $\vec{PR} = \langle 1, -1, 1 \rangle$ since they both lie in the plane through P, Q, and R. So,

$$\vec{PQ} \times \vec{PR} = \langle (2)(1) - (-1)(-1), (-1)(1) - (-3)(1), (-3)(-1) - (2)(1) \rangle = \langle 1, 2, 1 \rangle.$$

Therefore, $\langle 1, 2, 1 \rangle$ (and any scalar multiple thereof) is orthogonal to the plane through P, Q, and R.

(b) Find the area of the triangle PQR.

The area of $\triangle PQR$ is half of the area of the parallelogram determined by the three points. From (a), the area of that parallelogram is

$$|\vec{PQ} \times \vec{PR}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6},$$

so the area of $\triangle PQR$ is $\frac{1}{2}\sqrt{6}$.

§12.5 #33 Find the equation of the plane through the points $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$.

The vectors

$$\vec{a} = \langle 8 - 3, 2 - (-1), 4 - 2 \rangle = \langle 5, 3, 2 \rangle$$

and

$$\vec{b} = \langle (-1) - 3, (-2) - (-1), (-3) - 2 \rangle = \langle -4, -1, -5 \rangle$$

lie in the plane, so a normal vector to the plane is

$$\vec{a} \times \vec{b} = \langle -15 + 2, -8 + 25, -5 + 12 \rangle = \langle -13, 17, 7 \rangle$$

and so the equation of the plane is (using the first point on the plane)

$$-13(x - 3) + 17[y - (-1)] + 7(z - 2) = 0,$$

or

$$-13x \ + \ 17y \ + \ 7z = -42.$$

§12.5 # 57 Setting $z = 0$, the equations of the two planes become $5x - 2y = 1$ and $4x + y = 6$. Solving these two equations gives $x = 1$, $y = 2$ so a point on the line of intersection is $(1, 2, 0)$. A vector v in the direction of this intersecting line is perpendicular to the normal vectors of both planes. So we can use

$$\vec{v} = \vec{n_1} \times \vec{n_2} = \langle 5, -2, -2 \rangle \times \langle 4, 1, 1 \rangle = \langle 0, -13, 13 \rangle.$$

or equivalently we can take

$$\vec{v} = \langle 0, -1, 1 \rangle$$

and symmetric equations for the line are

$$x = 1, \frac{y - 2}{-1} = \frac{z}{1}$$

or $x = 1$, $y - 2 = -z$.

§12.6 #21-28 Match the equation with its graph (labeled I-VIII). Give **Reasons** for your choices.

There are many ways to solve this set of problems. You could look at the traces of each equation by letting $y = k$. This will immediately eliminate several choices for each equation. By doing this again to the x and z variables, you will know exactly what the traces in each of your three chosen planes are and can choose appropriately from there. The correct choices are

21. VII
22. IV
23. II
24. III
25. VI
26. I
27. VIII
28. V