Homework 1 solutions

12.1 #16 Find the center and radius of

$$x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0.$$

Completing the square in x and y gives

$$(x^2 + 8x + 16) + (y^2 - 6y + 9) + (z^2 + 2z + 1) = -17 + 16 + 9 + 1,$$

$$(x + 4)^2 + (y - 3)^2 + (z + 1)^2 = 9$$

which is a sphere of radius 3 with center $(-4, 3, -1)$.

12.2 # 32 Find the tension (vector) in each wire and the magnitude of each tension.

Call the two tensile forces $\vec{T_3}$ and $\vec{T_5}$ corresponding to the ropes of length 3 m and 5 m. In terms of vertical and horizontal components,

$\vec{T_3} = -|\vec{T_3}|\cos(52^o)\vec{i} + |\vec{T_3}|\sin(52^o)\vec{j}$ (1)

and

$\vec{T_5} = |\vec{T_5}|\cos(40^o)\vec{i} + |\vec{T_5}|\sin(40^o)\vec{j}.$ (2)

The resultant of these forces, $\vec{T_3} + \vec{T_5}$, counterbalances the force of gravity acting on the decoration [which is $-5g\vec{j} \approx -5(9.8)\vec{j} = -49\vec{j}$]. So, $\vec{T_3} + \vec{T_5} = 49\vec{j}$. Hence, using (1) and (2),

$$\vec{T_3} + \vec{T_5} = (-|\vec{T_3}|\cos(52^o) + |\vec{T_5}|\cos(40^o))\vec{i} + (|\vec{T_3}|\sin(52^o) + |\vec{T_5}|\sin(40^o))\vec{j} = 49\vec{j}.$$

Equating the components from above, we have

$$-|\vec{T_3}|\cos(52^o) + |\vec{T_5}|\cos(40^o) = 0, \qquad \text{and} \qquad |\vec{T_3}|\sin(52^o) + |\vec{T_5}|\sin(40^o) = 49.$$

From the first of these two equations,

$$|\vec{T_3}| = |\vec{T_5}|\frac{\cos(40^o)}{\cos(52^o)}.$$

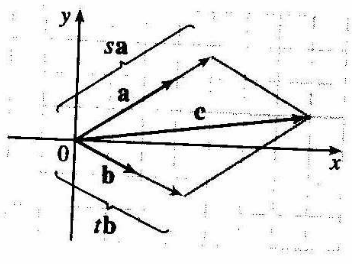
Substituting this into the second equation gives

$$|\vec{T_5}| = \frac{49}{\cos(40^o)\tan(52^o) + \sin(40^o)} \approx 30\,N \;\Rightarrow |\vec{T_3}| \approx 38\,N.$$

Finally, from (1) and (2), $\vec{T_3} \approx -23\vec{\imath} + 30\vec{\jmath}$ and $\vec{T_5} \approx 23\vec{\imath} + 19\vec{\jmath}$.

12.2 # 39

(a), (b)



(c) From the sketch, we estimate that $s \approx 1.3$ and $t \approx 1.6$.

(d) $\mathbf{c} = s\,\mathbf{a} + t\,\mathbf{b} \quad \Leftrightarrow \quad 7 = 3s + 2t$ and $1 = 2s - t$.
Solving these equations gives $s = \frac{9}{7}$ and $t = \frac{11}{7}$.

Figure 1:

12.3 # 22 Find, correct to the nearest degree, the three angles of the triangle with vertices $D = (0, 1, 1)$, $E(-2, 4, 3)$, and $F(1, 2, -1)$.

Let d, e, and f be the angles at vertices D, E, and F, respectively. Then d is the angle between vectors $\vec{DE}$ and $\vec{DF}$, e is the angle between vectors $\vec{ED}$ and $\vec{EF}$, and f is the angle between vectors $\vec{FD}$ and $\vec{FE}$. Thus, we have the following:

$$\cos(d) = \frac{\vec{DE}\cdot\vec{DF}}{|\vec{DE}||\vec{DF}|} = \frac{\langle -2, 3, 2\rangle \cdot \langle 1, 1, -2\rangle}{\sqrt{17}\sqrt{6}} = \frac{-3}{\sqrt{102}},$$

so

$$d = \cos^{-1}\frac{-3}{\sqrt{102}} \approx 107^o;$$

$$\cos(e) = \frac{\vec{ED}\cdot\vec{EF}}{|\vec{ED}||\vec{EF}|} = \frac{\langle 2, -3, -2\rangle \cdot \langle 3, -2, -4\rangle}{\sqrt{17}\sqrt{29}} = \frac{20}{\sqrt{493}},$$

so

$$e = \cos^{-1}\frac{20}{\sqrt{493}} \approx 26^o;$$

$$\cos(f) = \frac{\vec{FD}\cdot\vec{FE}}{|\vec{FD}||\vec{FE}|} = \frac{\langle -1, -1, 2\rangle \cdot \langle -3, 2, 4\rangle}{\sqrt{6}\sqrt{29}} = \frac{9}{\sqrt{174}},$$

so

$$f = \cos^{-1}\frac{9}{\sqrt{174}} \approx 47^o.$$

An alternate solution would be to apply the Law of Cosines three times as follows:

$$\cos(d) = \frac{|\vec{EF}|^2 - |\vec{DE}|^2 - |\vec{DF}|^2}{2|\vec{DE}||\vec{DF}|},\ \cos(e) = \frac{|\vec{DF}|^2 - |\vec{DE}|^2 - |\vec{EF}|^2}{2|\vec{DE}||\vec{EF}|},$$

and

$$\cos(f) = \frac{|\vec{DE}|^2 - |\vec{EF}|^2 - |\vec{DF}|^2}{2|\vec{DE}||\vec{EF}|}.$$

12.3 # 36 Find the scalar and vector projections of $\vec{b}$ onto $\vec{a}$.
First, $|\vec{a}| = \sqrt{1+4} = \sqrt{5}$. The scalar projection of $\vec{b}$ onto $\vec{a}$ is

$$comp_a b = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{-(1)(4)+(2)(1)}{\sqrt{5}} = \frac{-2}{\sqrt{5}}.$$

The vector projection of $\vec{b}$ onto $\vec{a}$ is

$$proj_a b = \frac{-2}{\sqrt{5}}\frac{\vec{a}}{|\vec{a}|} = \frac{-2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\langle 1,2\rangle = \frac{-2}{5}\langle 1,2\rangle = \langle\frac{-2}{5},\frac{-4}{5}\rangle.$$