Seeing Better: Optimizing Surgically Induced Astigmatism Correction Factors for Cataract Surgery*

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Abstract: Cataract surgery is one of the most common operations performed in the United States each year [1]. Cataract surgery is seen as a ‘routine’ procedure; however, it is not without complication. The surgery changes the natural astigmatism of the eye, which can cause blurry vision. It is possible to correct for this surgically induced astigmatism (SIA) during the operation. Currently, a standard correction factor of 0.5 diopters (D) is used. While this correction factor produces respectable results, we endeavored to improve upon the model and deliver individualized SIA predictions. The goal of this article is to predict the SIA for use in surgery on a second eye, based upon prior surgical results from the same patient. Pre-operative and post-operative cataract surgery data was gathered from a private ophthalmology practice. SIA values were then calculated, and a two-sample t-test was done to compare the mean values for left and right eyes. When no significant difference was found, we performed regression analysis to determine a model for SIA prediction. We analyzed our models using residual plots, a chi-squared test for goodness of fit, and generated a graphical comparison between our model and the standard correction factor of 0.5D. We found that our model is on par with the currently

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accepted standard correction factor and is, in fact, more accurate in cases of significant difference between the models. We believe we have generated a useful tool that can be easily and successfully utilized in patient care.

**Introduction**

A cataract is an eye disease that involves the clouding or opacification of the natural lens of the eye [2]. Cataracts cause blurry and/or discolored vision. Left untreated, they can cause blindness. This deterioration in vision inhibits normal daily activities such as reading and driving. Cataracts are very prevalent, and it is estimated that 20.5 million Americans (17.2%) over the age of 40 have cataracts in either eye [3]; however, cataracts can be removed, restoring vision, through a routine procedure. Due to the drastic improvement in vision and high success rate, cataract surgery is one of the most common operations performed in the United States [1].

While cataract surgery has incredible benefits, it is not without complication. One of the effects of cataract surgery is to change the natural astigmatism of the eye. Astigmatism is a condition caused by an unevenly shaped cornea that results in different focal points for light entering the eye at different angles. Astigmatism is characterized by two separate but related measurements. To calculate astigmatism the eye is measured with a keratometer, which gives readings of the radius of curvature of the cornea of the eye. The first reading taken consists of two parts: the measurement that gives the steepest (smallest) radius of curvature and the meridian of that measurement (the axis at which the measurement is taken). The second reading is the radius of curvature of the eye at ninety degrees to the first measurement.

![Diagram of light passing through the cornea at different angles](image)

Figure 1: Diagram of light passing through the cornea at different angles [4]

Astigmatism, measured in diopters, is the difference between these measurements. A diopter is a unit of measurement that describes the power of
a lens. It is equal to the reciprocal of the focal length, measured in meters. For example, a 4 diopter lens brings parallel rays of light to focus in 1/4 or 0.25 meters. Therefore, the greater the K reading, the larger the diopter of the lens, and the closer the focal point of light entering the eye is to the lens. For example, if the eye is ovoid in shape, with the smaller radius being in the horizontal plane (as pictured Fig. 1), then the first reading (the steep K) could be 44.0 D, and the meridian, the angle of the plane, would be 0°. The second reading (the flat K), would be taken at 90° to the plane of the steep K reading, and could be 43.0 D. In this case the astigmatism would be |44.0 - 43.0| = 1.0 diopter (D).

There is an incision made in the eye during cataract surgery through which the old opaque/distorted lens is broken up and removed, and the new intraocular lens (IOL) is implanted. This incision alters the shape of the eye, which changes the eye’s natural astigmatism. This change in astigmatism due to cataract surgery is called surgically induced astigmatism (SIA).

Unfortunately, it is not possible to simply calculate both the pre-operative and post-operative astigmatism by the above method, and use the difference in the two to find the SIA. Calculating the SIA is complicated by the fact that the meridians of the K values also change. For example, both the steep K and flat K could be shifted 35 degrees clockwise from their original readings, and thus vector analysis is necessary to calculate the SIA. Our SIA calculations were done using an SIA calculator created by Warren Hill, MD.

With the implantation of a new lens it is possible to correct not only for pre-existing astigmatism, but for surgically induced astigmatism as well. The Toric lens is a lens that corrects for astigmatism, and is a relatively new development in the field of IOL correction. The very first Toric IOL was approved by the FDA in 1998, and the more recent Atom Toric lens was just approved in September of 2005, [3]. The Toric lens is expensive, costing an additional $500 - $800 beyond the cost of the cataract surgery, and is not covered by most insurance providers. This new and costly technology has not yet been perfected, and thus lends itself perfectly to optimization studies.

The problem in correcting for SIA, along with previous astigmatism, is estimating how much the incision will change the shape of the eye (and thus predicting the value of the SIA). The generally accepted SIA correction - the amount that the physician varies the power of the implanted lens from correcting for just the previous natural astigmatism - is 0.5D. If it was possible to generate a more accurate correction factor, or to predict the SIA before surgery, then a more correct lens could be selected for the patient, leading to greater clarity of vision.

The goal of this work is to determine, with some level of confidence, the relationship of surgically induced astigmatism between two eyes of the same patient. Based on this relationship, our aim is to create a model for the prediction of SIA values for a patient’s second cataract surgery. This advancement would yield further optimization for patients that undergo IOL implantation in both eyes.

We began by calculating the surgically induced astigmatism for a patient data set containing 17 patients who had operations performed on both left (OS)
and right (OD) eyes. We disregarded data for those patients who had surgery on only a single eye. Eyes with previous surgery or scarring were omitted. After calculating the SIA for all patients, we performed a two sample t-test to determine whether or not mean values of SIA for left and right eyes were significantly different, enabling us to generate a standard SIA correction value.

We then performed regression analysis using various models including linear, power and logarithmic functions. Regressions were performed for left eye vs. right eye. After determining which of our models was most accurate we also performed right eye vs. left eye analysis, as well as a random mix of left and right eyes on both axes. The additional analysis was done to derive formulas for predicting the SIA of a patient who had one previous cataract surgery, regardless of which eye was operated on.

After completing the regression models, we calculated the residuals for both left and right eyes. We also performed a Chi-Squared test for goodness of fit on the left and right eye power regression models, as well as testing the standard correction factor for comparison. As a visual representation of the differences between the results of our models and the standard correction factor of 0.5D, we constructed bar graphs displaying both the difference between actual SIA and our model as well as the difference between actual SIA and the standard correction factor.

Data

Patient data were obtained from Eye Associates of Marquette, a private ophthalmology practice. The practice provided pre- and post-operative data from recent cataract surgeries. Of the data provided, we selected only the patients who had surgery performed on both eyes for the computation of our models. Also, data were disregarded for those patients who underwent previous surgeries, or had non-ideal corneas (Ex. those patients with corneal scarring). In this way we were able to screen possible outliers and surgical data that could skew the response distribution. After selection of candidates for analysis, SIA calculation was done using a spreadsheet calculator obtained from the website of Dr. Warren Hill and East Valley Ophthalmology [6].

We constructed histograms for left and right eye SIA values. A bin size of 0.1 was selected based upon the number of patients and the spread of data.

Statistical Analysis:

After visualizing the SIA data we performed a two-sample t-test for determining whether the means of the two populations, right eye SIA ($\mu_1$) and left eye SIA ($\mu_2$), were significantly different (Null hypothesis $H_0 : \mu_1 = \mu_2$, Alternative hypothesis $H_a : \mu_1 \neq \mu_2$). The t-test was best suited for our data because it fit the assumptions of the test, which are small sample size, no extreme outliers, and a non-normal distribution.

Our test statistic was $-0.0198$, yielding a two-tailed $P$-value of 0.9844. For
Table 1: Pre-operative and post-operative keratometric data, including meridians, used in the vector analysis calculations of surgically induced astigmatism. Calculations were performed for both left and right eyes of 17 patients.

<table>
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<th>Pre-op Flat K</th>
<th>Pre-op Steep Median</th>
<th>Pre-op Flat Median</th>
<th>Post-op Steep K</th>
<th>Post-op Flat K</th>
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Figure 2: Histogram of Surgically Induced Astigmatism of the right eye. Distribution of SIA measures is roughly symmetric although not normally distributed, with no extreme outliers.

For an alpha of .05, the test was not significant, and we found no statistical difference between the mean values of the SIA of the left and right eyes.

Regression analysis was performed successively using five different models:

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linear, polynomial, logarithmic, exponential, and power. Each model was computed using left eye vs. right eye SIA. Upon determination of the model with the greatest $R^2$ value (the power regression), we also performed the same regression for right eye vs. left eye, as well as a randomized sample group. We randomly split the patient set into two groups, using right eye data of one group and left eye data of the other as the independent variable. The other eye of each patient was plotted as the dependent variable.

Figure 4: Bar graph comparing the amount of error of the power model to the standard correction factor for the left eye.

We created residual plots of the SIA data using our power model, allowing
Figure 5: Bar graph comparing the amount of error of the power model to the standard correction factor for the right eye.

Figure 6: Equation for line of best fit for logarithmic regression model: $y = 0.2023 \ln(x) + 0.6508$. $R^2$ value of 0.3143

visualization of the accuracy of our model. Separate plots were created for right and left eyes. Both plots show a similar distribution of residuals.

Further comparison between our models and the standard correction factor model was done with a chi-squared goodness of fit test. Our null hypothesis (H0) for each model was that our values predicted by the power models were not significantly different than the actual values. The alternative hypothesis (Ha) was that there was a significant difference between our predictions and the actual results. Similar hypotheses were formed for testing the standard
Figure 7: Equation for line of best fit for exponential regression model: \( y = 0.2107e^{1.3831x} \). \( R^2 \) value of 0.2644.

Figure 8: Equation for line of best fit for power regression model: \( y = 0.7232x^{0.6415} \). \( R^2 \) value of 0.403.

correction model of 0.5D. For the right eye power model, a chi-squared statistic of 1.7913 was found, giving a p-value of approximately 1.0 with 16 degrees of freedom. For the left eye power model, a chi-squared statistic of 1.4012 was found, giving a p-value of approximately 1.0 with 16 degrees of freedom. For the left and right eye standard correction factor of 0.5, the chi-squared statistics were 1.6767 and 1.8156. Both had p-values of approximately 1.0. Both have 16 degrees of freedom.

In an effort to compare our power model to the accepted SIA correction factor of 0.5D, we constructed bar graphs displaying their accuracies. Graphs
Figure 9: Equation for line of best fit for power regression model: \( y = 0.7053x^{0.6282} \). \( R^2 \) value of 0.403.

Figure 10: Equation for line of best fit for power regression model: \( y = 0.6956x^{0.6806} \). \( R^2 \) value of 0.4135.

display the difference between actual SIA and both predicted SIA and the standard correction factor. A smaller value for the difference between the actual SIA and the respective model indicates a better fit.

Conclusions:

The methods for performing cataract surgery have been improving for many years and have reached a point where the surgery itself follows a routine pro-
Figure 11: Residual plot of right eye SIA power regression model used to predict left eye SIA.

Figure 12: Residual plot of right eye SIA power regression model used to predict left eye SIA.

cedure. While the procedures are straightforward, predicting the changes that will occur in the eye as a result of the surgery is not a perfect science. Doctors currently use a standard correction factor, typically 0.5D, as a prediction of the surgically induced astigmatism.

Our intention with this project was to improve this common practice and help predict the SIA on an individualized basis, based on prior surgical results. We theorized that this would be possible based on preliminary investigation comparing the SIA means of patients’ left and right eyes with a two-sample t-test. The t-test showed no statistically significant difference between the population means of the two eyes. This led us to believe that there was a correlation between the SIA of two eyes of the same patient. We then looked for a model to represent the relationship between the two eyes of the same
Figure 13: Bar graph comparing the amount of error of the power model to the standard correction factor for the left eye.

Figure 14: Bar graph comparing the amount of error of the power model to the standard correction factor for the right eye.

patient.

Of all the regression models that we tested, the power model proved to
have the best fit. The value of the coefficient of determination for the non-
normalized model is 0.403. While this is not an ideal value, further testing
proved the validity of our model. Our randomized model, a mix of the left
and right eyes on both axes, had a coefficient of determination value of 0.4135.
This further suggests that the classifying of left and right eyes is arbitrary for
this study.

We created residual plots of both left and right eye power regression models
for a visual representation of their accuracy. The plots appear similar, with no extreme outliers, and the majority of data points fell within 0.3D of the x-axis.
There seems to be no definite trend to the residual plots, thus they do not suggest a different and more accurate model. We also performed a chi-squared test for goodness of fit on the two regression models. Based on the p-values, we
do not reject our null hypothesis, and can conclude that there is no significant
difference between the values predicted by our model and the actual SIA values.
We also calculated the chi-squared statistics for the standard correction model
of 0.5D. For both left and right eyes, our models had slightly better chi-squared
statistics. This shows that our model, while not a perfect representation of the
given data, is at least as good as the currently accepted model of 0.5D.

Finally, we constructed bar graphs in order to compare predicted values
from our model and the correction factor of 0.5D to the actual SIA for individual
patients. Our power regression model gave a more accurate prediction of SIA
than the correction factor of 0.5D fifty percent of the time when combining left
and right eye data. Furthermore, in 9 of the 15 cases when the difference in
accuracy of the two models was greater than approximately 0.1D, our model
gave a better prediction and was closer to the actual SIA. This shows that it is
useful to have an adaptive model that can account for SIA values different than
0.5D. While the standard correction of 0.5D accurately represents the mean SIA
values, it does not represent the individual variance from patient to patient.
As a constant, it cannot accurately represent the range of induced astigmatism
found after surgery. The power model provides flexibility and adaptability for
those patients whose prior surgical results display differences from the norm.

We believe that we have found a useful tool for doctors to apply in the
calculation of a surgically induced astigmatism correction factor for cataract
surgery. This method can easily be applied in a clinical setting as is; however,
further study should be done. Difficulties in obtaining data for patients
who had surgery performed on both eyes led to a small patient sample for this
study. Adding more patient data may improve the prediction model and in-
crease the accuracy of predictions for patients whose SIA varies from 0.5D. One
drawback of this method is that it can only be used to predict SIA values for
a second cataract surgery performed on the same patient’s second eye. With
further study, it may be possible in the future to predict SIA values based on
keratometric measurements alone, and not rely on previous surgical results. At
the present, the power regression model provides a useful alternative to the
currently accepted SIA correction factor and has the potential to be a power-
ful tool for doctors when performing a routine but necessary surgery for the
restoration of sight.

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References


